Possibilistic Resource Allocation Optimization Models for Improving Access Safety over a Network of Interurban Highways

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Abstract
Due to the importance of the access safety management, both from social and from engineering perspective, but also due to the scale of the investments that usually are necessary for improving road safety, optimal safety management strategies -for decades now- has an elevated importance in transportation planning research and within the industry/authorities community. In the current study, optimal strategies of budget allocation for network safety improvement are identified by formulating and solving a possibilistic combinatorial programming problem for a pre-defined set of 'black' spots. Specifically, a resource allocation programming problem is constructed, aiming on selecting the locations subset such as to maximize total network safety performance. For treating the problem's uncertainty the proposed formulation adopts concepts from fuzzy sets theory by assigning a fuzzy set for the safety improvement in each location with respect to the type of improvement that is required in each case. This possibilistic problem setup is tackled by suitable transformations of two hybrid optimization routines, namely, Genetic Algorithms and Simulated Annealing. Results from the proposed framework implementation onto a network of realistic characteristics provides evidence on its performance in real-world cases.

Keywords: *Access and Mobility Management; Highways Networks Safety; Optimal Budget Allocation; Fuzzy Combinatorial Programming; Evolutionary Optimization*

1. INTRODUCTION

Effective access and mobility management strategies involve several actions related to efficiency, sustainability and safety. Especially in road transportation, safety improvement programming and scheduling is a constant major concern of highway agencies. The involvement of human lives, gives to the related problem a tremendous significance demanding special attention. In identifying this point, a great part of the public funding in the countries of EU and elsewhere is directed towards the maintenance and improvement of the conditions of highways, in an effort to reduce human casualties by increasing the level of public safety (Highway Safety Manual, 2010). Optimal allocation of funds for highway safety improvements is a complicated and often tedious task, involving performance maximization of the invested capital with respect to the reduction of vehicle accidents and fatalities. In large-scale paradigms of network management (i.e. country-wide highway systems), the number of competing alternative improvement programs can be extremely large, requiring a methodology for prioritizing the recommended projects, such that the return from the utilization of a limited budget is maximized. At the same time, the prioritizing mechanism is subject to the stochastic nature of accidents occurrence within a network of highways.

In the current study, optimal strategies of budget allocation for network safety improvement are identified by formulating and solving a combinatorial dynamic programming problem for a pre-defined set of 'black-spots' (locations that the concentration of accidents is occurred over an annual period). Specifically, a resource allocation programming problem is constructed, by selecting the location and the type of improvements that will be made in each period, aiming to maximize total network safety performance. For treating stochasticity, the problem is formulated by adopting concepts from fuzzy set theory and in particular by assigning a fuzzy set for the safety improvement in each location with respect to the type of improvement that is required in each case. This possibilistic problem setup is tackled by suitable transformations of a suite of hybrid optimization routines, namely, Genetic Algorithms and Simulated Annealing. The results of the multiple runs allow the comparative analysis of their computational performance.

The application of the proposed framework is tested over a realistic large-scale test-bed from the Greek interurban highway network, illustrating the applicability of the above framework to similar cases of safe road access and mobility management. The results are compared also with those of the deterministic problem setup, providing useful information on the role and the importance of uncertainty in budget allocation cases, while remarks on the value of reliable programming of tasks are providing, extending the importance of such type of analysis to other cases of transportation-related investments scheduling.

2. BACKGROUND REVIEW

Budget allocation and actions ranking for road safety, as an area that effort could easily lead to boondoggle projects, has been identified as an important element in transport planning (AASHTO 2010, Gross et al. 2010). Especially in cases of access management towards road safety of existing inter-urban networks, where the severity of accidents, the capital involved in interventions over a realistic nation-wide network (bearing in mind the scale of relevant project) and the scarcity of resources for upgrading road infrastructure, the importance is elevated for a well-selected methodology for identifying as well as prioritizing the spots and the type of interventions that should be prioritized. From the typical scheme for safety improvement ('black spots' identification, alternative measures identification/economic appraisal for each of them and selection/prioritization of funds), the current study is focusing on the final part of the process. The background review of the large body of literature that exists, will not be in any sense exhaustive due to space limitations, but will cover the most relevant publications of the specific subject.

For the above reasons, highways road safety has been an area of increased research interest for a long time. A brief review of the research conducted so well as for an outlook of the foundations of the causal analysis of road accidents can be found in Spring (2005), while on the highway inspections (audits) related to identification of possible hazardous ('black') spots some issues can be found in Kanellaidis (2000). Moreover, over a large number of methods available on the selection/characterization of a particular section (location) as 'black' spot, all of them are either based on a Bayesian/probabilistic framework (e.g. Saccomanno et al. 2001, Miranda-Moreno and Fu, 2006), or on rule/knowledge-based systems (e.g. Chassiakos et al. 2005). From the above studies, although indicative, two issues are introduced: the random/uncertain nature of the accidents occurrence and the probabilistic characterization of a particular section (or location) as a 'black' spot.

Moving to the performance evaluation of invested capital for the safety improvements, the results of before-and-after analysis (Al-Masaeid 1997, Lin et al. 2003, Mauro and Cattani 2004) can suggest that the accidents reduction rate is also a random variable for various improvement scenarios. Moreover, it is regarded as state-of-the-practice (e.g. Golob et al. 2004) that road safety-oriented improvements can also have some effect of the operational characteristics of highway sections (traffic flow, speed, driver attitude, etc), an element that could have a consequent effect on road safety, or locally, either in neighboring locations. This element is also important when dealing with network-wide safety management problems.

In the current study, the above uncertainty issues is aimed to be treated within a possibilistic selection mechanism of 'black' spots for allocating restricted budgetary resources in a network of inter-urban highways by identifying fuzziness in the rate of improvement, in terms of accidents, fatalities or injuries saved after the implementation of specific infrastructure upgrading actions. On the next section, starting from a standard deterministic formulation of an optimal 'black' spot selection mechanism, appropriate modifications for incorporating uncertainty are presented and analyzed. The selection mechanism is then based on fuzzy integer mathematical programming theory. Also, the solution approach is briefly presented which is based on suitably hybridization of Genetic Algorithms (GA).

3. 'FUZZY' FORMULATION AND GENETIC SOLUTION OF THE OPTIMAL SAFETY BUDGET ALLOCATION PROBLEM

Starting from the point where for a system of highways a set of 'black' spots have been identified, the causal analysis is performed, upgrading actions listed and economically appraised, in cases of budgetary limitations the next step is the selection of a subset of them for optimizing capital investment (in terms of safety improvement). This case here is treated as an optimal subset selection process among distinguished actions for maximizing the results of an 'effort' (here capital investment). In this study the proposed methodology corresponds to a suitable adaptation of the integer/binary programming selection process, also termed as 'the Knapsack Problem'(Papadimitriou and Steiglitz, 1998), to the complete set of 'black' spots for selecting, ideally, the most 'prosperous' subset.

In particular, given a set I of candidate 'black' spots i ($i \in I$) in which a_i accidents occurs annually, with f_i fatalities (with f^C€ marginal cost) and g_i injuries (with g^C€ marginal cost) and that for each i the improvement cost is c_i, while the total available budget is B. Then given the binary variable $x_i \in \{0,1\}$ (if location i is selected $i = 1$ otherwise, $i = 0$), an optimal selection corresponds to the solution ($x_i^*, \forall i \in I$) of the following binary programming problem:

$$\max_x Z = \sum_I x_i \left(f_i f^C + g_i g^C \right) \quad (1)$$

subject to:

$$\sum_I x_i c_i \le B, \text{ for } i \in I \quad (2)$$

Since the selection of a location i correspond to a 'gain' (reduction of the accident cost), the maximization of the objective function stands for the performance maximization of the available invested capital introduced in the constraints set. In cases where both binary selection of 'black' spots as well as the degree of an implementation (expressed as continuous variable) is sought, then the case can be handled as a mixed-binary optimization problem by introducing a degree of implementation continuous variable (Melachrinoudis et al. 2002), while Karlaftis et al. (2007) provide a similar application of budget allocation problems in discrete transportation-related locations (bridges).

It is noted that alternative studies (PIARC 2007, ERF 2002, ROSPA 2011) have provided different measures-of-effectiveness for each type of infrastructure upgrading action (Table 1).

Table 1. *Estimation for some indicative actions of the measure-of-effectiveness (accidents reduction percentage) by 3 alternative organizations.*

Intervention/Action	PIARC	ERF	ROSPA
Junction Improvements	73	88	44
Intersection Design	69	68	64
Anti-skid	69	62	57
Markings	29	39	34
Markings and Signs	27	39	41

From the above data two issues are evident: no action (at least, single action) implementation is able to ensure that the accident risk will be eliminated, and, the reduction rate is subject to local conditions and random effects and thus no accurate risk reduction rate estimation can be regarded.

If uncertainty is taken into consideration to the above optimization problem, fuzzy set theory may be utilized (Bellman and Zadeh 1970, Zimmermann 1978, Teodorovic and Vucadinovic 1998). In particular, the reduction rate of fatalities and injuries (in monetary values) after implementing the proposed set of infrastructure upgrading actions (of cost c_i) in each spot i, can be introduced by the following transformation:

$$\max_x G = \sum_I x_i \left(\mathrm{RF} f_i f^C + \mathrm{RI} g_i g^C \right) \tag{3}$$

subject to:

$$\sum_I x_i c_i \leq B \text{, for } i \in I \tag{4}$$

where RF and RI corresponds to the coefficients of the fatalities and injuries reduction rate which are expressed in fuzzy arithmetic. Reminding basic concepts of fuzzy arithmetic used here, RF and RI is modeled as triangular fuzzy numbers (Fig. 1).

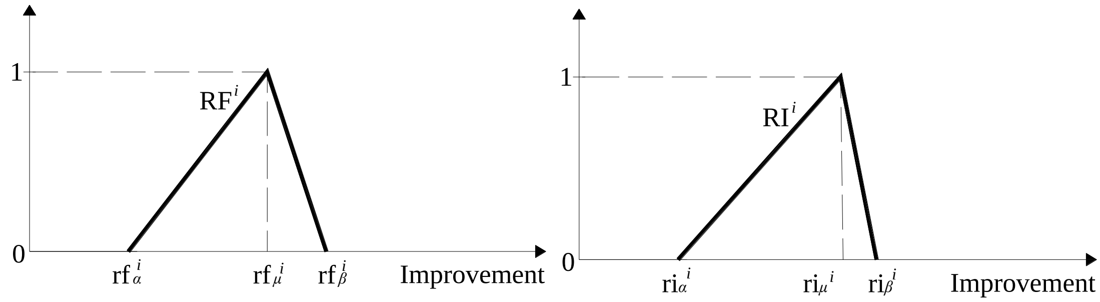

Figure 1. *Fatalities (RF) and Injuries (RI) Reduction Rate at Location* i*, Expressed in Membership Functions of Fuzzy Arithmetic.*

Without getting in much details due to space limitations, the multiplication and addition operations that composes the objective function G (Eq. 3), in fuzzy arithmetic now provides a set of 'possible' values instead of 'crisp' (single) values (Eq. 1). In particular, the set of possible fatalities total cost, FC, now is expressed by the fuzzy number:

$$FC = \left[\sum_i x_i rf_a^i f_i f^C, \sum_i x_i rf_\mu^i f_i f^C, \sum_i x_i rf_\beta^i f_i f^C \right] \tag{5}$$

and the injuries cost, IC, is correspondingly:

$$IC = \left[\sum_i x_i ri_a^i g_i g^C, \sum_i x_i ri_\mu^i g_i g^C, \sum_i x_i ri_\beta^i g_i g^C \right] \tag{6}$$

and total accidents cost TC:

$$TC = FC + IC = \begin{bmatrix} \sum_i x_i rf_a^i f_i f^C + \sum_i x_i ri_a^i g_i g^C, \\ \sum_i x_i rf_\mu^i f_i f^C + \sum_i x_i ri_\mu^i g_i g^C, \\ \sum_i x_i rf_\beta^i f_i f^C + \sum_i x_i ri_\beta^i g_i g^C \end{bmatrix} = \begin{bmatrix} fm_a^i, \\ fm_\mu^i, \\ fm_\beta^i \end{bmatrix} \tag{7}$$

For the current case, rf_β^i and ri_β^i stand for the total annual number of fatalities and injuries at each location i, while the left part of the fuzzy membership function expresses the belief about the reduction of fatalities and injuries that could be achieved by implementing the proposed actions at each spot i. As indicated in Table 1, different action plans could have different effects in terms of accidents severity.

After the above modifications on the nature and values of the objective function's components, it is straightforward that the objective function altogether is altered. The altered formulation is based here (among others available) on the concept of fuzzy sets termed 'Level-of-Satisfaction-LoS', $h \in [0,1]$. The LoS corresponds to the highest allowable degree that the upper/righter part of the membership function can be laid away from the most possible value of the membership function in order the solution to be acceptable/satisfying. After the introduction of the concept of LoS, for the triangular membership functions used in equation (7), the objective function G now can have the following form:

$$\max_x G = fm_\beta^i - \left(fm_\beta^i - fm_\mu^i\right)h \tag{8}$$

while the constraints set remains the same, since for the current problem setup no vagueness is assumed for these components:

$$\sum_I x_i c_i \le B, \text{ for } i \in I \tag{9}$$

The current formulation, since it aims to maximize the total benefits of the invested capital (in terms of reduction of total accidents cost) is also introducing the effects of the distance between the maximum and the most possible accident cost gains, an element that relates to the reliability of the selection set.

In terms of the complexity of the proposed possibilistic problem setup, is remains a combinatorial optimization problem and as so it belongs to the NP-hard class of problems. For addressing it and for comparative purposes, two population-based stochastic approximation algorithms have been utilized -in particular Genetic Algorithm (Goldberg 1989 and Simulated Annealing (Kirckpatrick 1983)- which have been extensively used in addressing such combinatorial problems, for their powerful capabilities in such circumstances. Due to space limitations, details on the coding of combinatorial problems will be omitted here since they can be found in relevant papers and textbooks.

4. COMPUTATIONAL EXPERIENCE

For gaining some insights on the performance of the proposed optimal budget allocation framework, the methodology has been applied to a realistic testbed. In particular, a set of 'black' spots covering 6 parts of the Greek national highway network that have been characterized in last years of elevated risk exposure (Fig. 2). These parts are the Thessaloniki-Kavala (No.1), Maliakos Gulf (No.2), Agrinio-Ioannina (No.3), Korinthos-Patra (No.4), Patra-Pyrgos (No.5) and Northern Crete axis (No.6) sections. The data have been obtained from the Greek Ministry of Infrastructure, Transport and Networks and correspond to the year 2005. As so, their retaining significance relates only to demonstration purposes, since for almost 3 years now significant upgrading projects are on progress altering the current road safety conditions from those exhibited in the dataset used here.

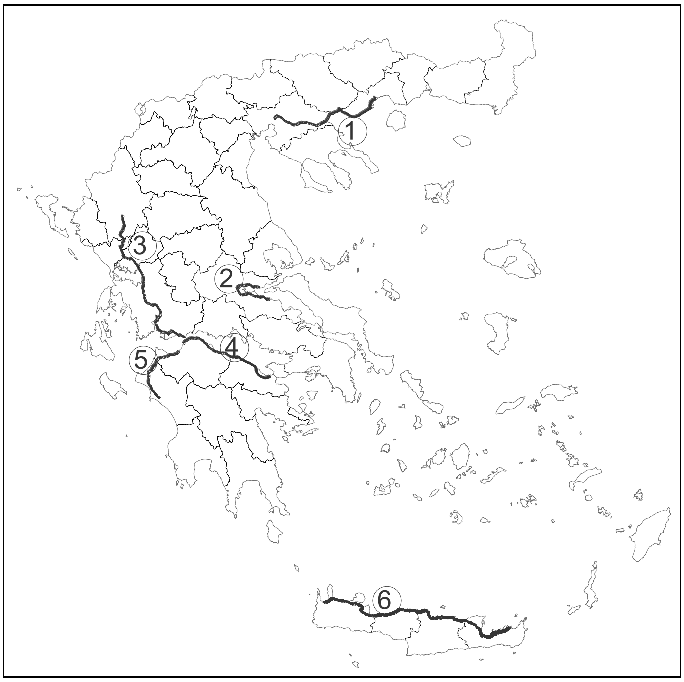

Figure 2. *Highway stretches where the used set of 'black' spots are located.*

It is noted that the 6 parts of the national highway network are scattered all over Greece, suggesting different construction periods (different materials, construction methods, regulations etc) and local operating conditions (loading patterns, weather, etc). Thus, despite the fact that the risk exposure may be similar, the necessary actions may be much different among alternative locations and highway stretch. In Figure 3, the values of the variables for the complete set of the 139 'black' spots are presented (for brevity) in color coding.

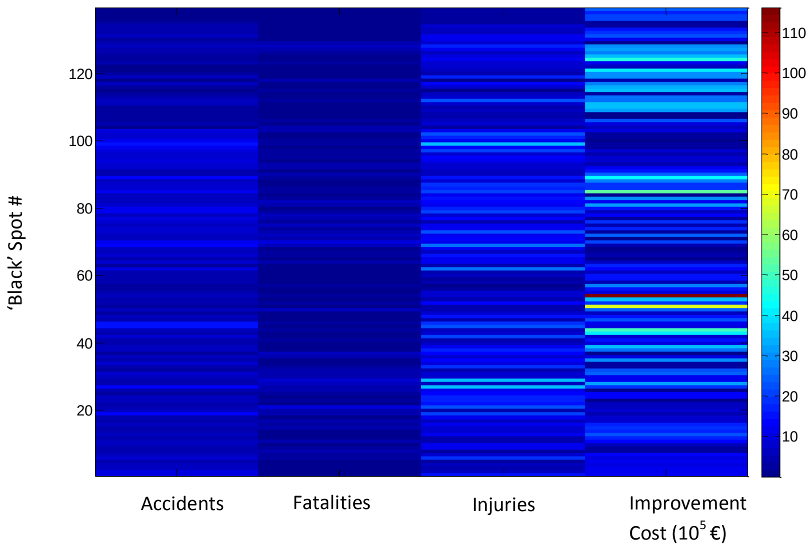

Figure 3. *Dataset, containing the number of Accidents, Fatalities, Injuries and the Economic Appraisal for each of the 139 'black' spots.*

The gross statistics of the testbed used here corresponds to 836 accidents, 295 fatally injured and 1516 seriously injured, while the total economic appraisal for safety improvements is 23247442 €. The marginal costs per fatality is taken as $f^C = 612141$€/person killed while for injuries $g^C = 467703$€/person injured (Yannis et al. 2005). It should be also mentioned that the corresponding triangular membership functions of the fuzzy numbers representing fatalities reduction rates for each implemented action has the following representative values $RF^i = [rf_a^i, rf_\mu^i, rf_\beta^i] = [0.8, 0.9, 1.0]$ while for injuries $RI^i = [ri_a^i, ri_\mu^i, ri_\beta^i] = [0.5, 0.7, 0.9]$ (see Fig. 1) expressing an assumption that after the improvement actions implementation the number of fatalities will more drastically be reduced than the overall number of injuries. Also a Level-of-Satisfaction/LoS of $h = 5\%$ has been selected. Finally, the total annual available budget for safety improvement projects is taken as $B = 15000000$€.

Further, in Figure 4 the plot matrix of correlation diagrams and variable distributions is provided, for getting some insight on the nature of the variables and thus the necessity of the possibilistic combinatorial problem that is proposed here.

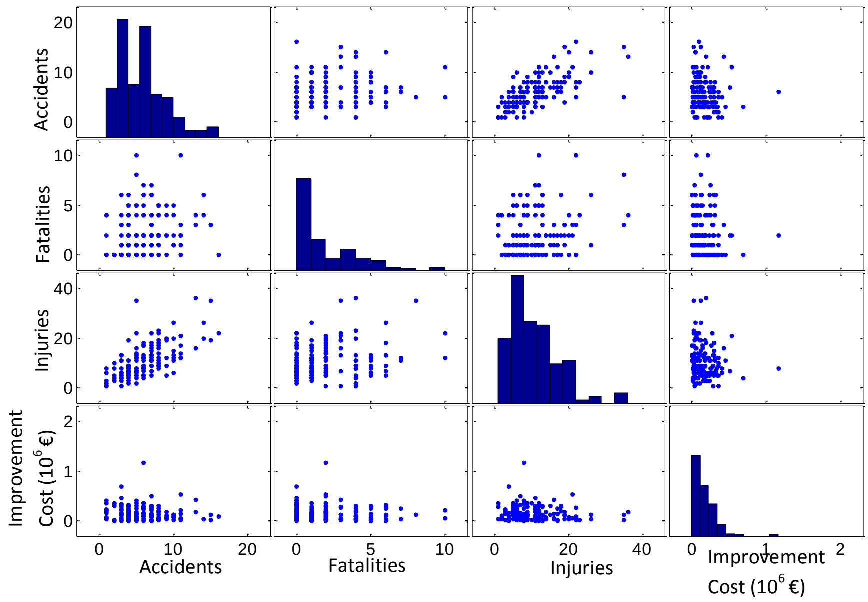

Figure 4. *Scatter and Distribution Plots Matrix of No. of Accidents, Fatalities, Injuries and Infrastructure Improvements Economic Appraisal.*

This analysis exposes the uncertainty involved in the particular problem, some of which is introduced by the random/unexplained cause of the accidents occurrence phenomenon and thus the subjectivity involved in the engineering judgment for the necessary actions that should be taken for each 'black' spot (resulting in much different actions for each case). These two elements leads to an overall uncertainty of the invested capital's performance in terms of access safety improvement, an element that has been highlighted in various studies (e.g. in Yannis et al. 2008). Regarding the properties of the 'optimal' subset selection problem, this corresponds to a binary fuzzy optimization problem of 2^{139} combinations (subject to constraints). As the scale of the combinatorial problem suggests, such circumstance cannot be addressed by enumeration and thus the above presented hybrid algorithms will be used. Without getting in technicalities for brevity, the setup of the Genetic Algorithm used for addressing the current problem setup corresponds to a population of 50 individual solutions, 10% Elitistic movement, 80% Crossover and 5% Mutation rates respectively. For the Simulated Annealing algorithm, starting from a random initial state, 50 function evaluations (produced by Boltzmann function) are accepted before 'Temperature' is updated (in exponential rate of $0.95^{Iteration\_No}$).

Initial the problem is addressed in its deterministic form (Eqs (1) and (2)), where the allocation of budget is based on selecting a subset of 'black' spots that maximizes benefits (are most 'costly' in terms of fatalities and injuries) within the predefined budget constraint. In Figure 5 the two representative algorithms convergence diagrams are provided, exposing their ability on addressing the current problem setup, but also providing some evidence on the quality of the solution (probability that optimal selection plan $x_i *$ is closely to the global optimal $X_i *$) since both converges to similar objective function values.

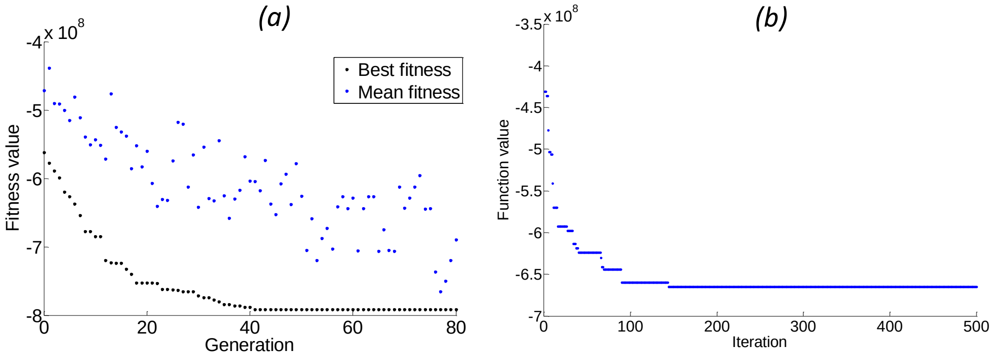

Figure 5. *Convergence Diagrams (a) for the Genetic Algorithms and (b) for the Simulated Annealing (the diagrams here show the plots of* $\min\{-Z\} \equiv \max Z$ *).*

Although the algorithms where able to come up with improved solutions (as compared with an initial random starting point), it can be observed that GAs performed better than SA, with $Z = 791855652$ € for almost the same number of objective function evaluations (computational time less than 3 mins).

For addressing the fuzzy optimization setup presented earlier, a GA with identical properties as the one used for the deterministic problem counterpart. A representative convergence diagram is provided in Figure 6, from which it can be observed that there is a slight reduction of the invested capital performance ($Z = 781669353$€) due to the role of the LoS.

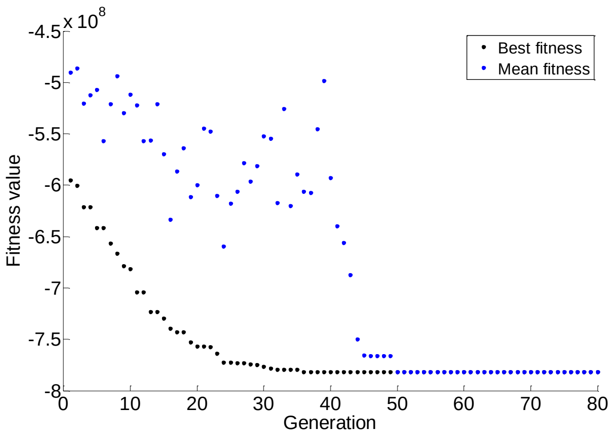

Figure 6. *Convergence Diagram of the Genetic Algorithm used for the Possibilistic Combinatorial Optimization Setup (the diagram here show the plot of* $\min\{-Z\} \equiv \max Z$ *).*

As it can be observed by the membership functions of the fatalities and injuries reduction rate and the formulation of the objective function, G, the selection of the optimal 'black' spots set is more favorable in incorporating locations with increased number of fatalities. Finally, the quality of the solution that leads towards to the selection of the spots with qualitative properties (here the more reliable fatal accidents reduction rate) can be regarded as an important feature in such decision support systems, since introduces reliability issues into a value-based selection mechanism.

5. CONCLUSIONS AND OUTLOOK

In the current paper, results from an exploratory analysis on the optimal budget allocation problem for improving access safety in highway networks were presented. The analysis is focusing on the properties of this important (social as well and engineering focused) issue, highlighting the role of scale as well as of the uncertainty involved in this specific problem. In the approach presented here, the selection mechanism of a subset of 'black' spots that optimize the invested capital performance (in terms of highways access safety improvement) is based on addressing a fuzzy combinatorial problem with stochastic approximation algorithms, namely Genetic Algorithms and Simulated Annealing. The presented approach is taking advantage of the powerful capabilities of these optimization algorithms in addressing large-scale problems but also incorporating the possibilistic features of the problem at hand into the 'optimal' 'black' spots selection.

Several points of outlook can be regarded for this line of research. At first, alternative fuzzy optimization setups can be investigated, introducing vagueness in the problem. For the fuzzy theoretical point of view, instead of the 'level of satisfaction', the concept of the 'Centre of Gravity' could be used as a metric for the 'optimal' fuzzy solution of the objective function. This extension may introduce a different approach in the possibilistic analysis used here closely, related to the concept of reliability in optimal interventions for road access safety management. Moreover, elements of fuzziness could be regarded in the constraints set too, a modification that will significantly alter the problem properties and thus its final solution. Also, useful results could come up (in computational effort and in the algorithmic performance) by the comparative analysis of the above presented possibilistic formulation of the problem with its probabilistic counterpart.

From the access safety policy point of view, since in recent years different authorities have responsibility for parts of the (inter-)national highway system, the effects of central vs. decentralized organization/management schemes in budget optimization can be quantitatively investigated within the proposed optimization framework, providing useful insights to the incentives and drawbacks of alternative management organization to highways access safety. Moreover, a realistic extrapolation of the proposed optimization framework could be its generalization to the multi-period case, which corresponds to a fuzzy combinatorial dynamic programming problem setup. Finally, the more detailed estimation of the reduction rates membership functions (per specific safety improvement action) will increase the accuracy of the results as well as its usefulness.

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